Closing Thurs: HW 9.6, 9.7(1)

Read HW hints in newsletter!!!!

Closing Tues: HW 9.7(2), 9.8, 9.9

Exam 1 is Thurs, Jan. 25th covers 9.4 - 9.9.

Derivative Rules: Sum, Coef. Rules and

$\frac{d}{dx}(x^n) = n\,x^{n-1}$.

$$\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + f'(x)g(x)$$

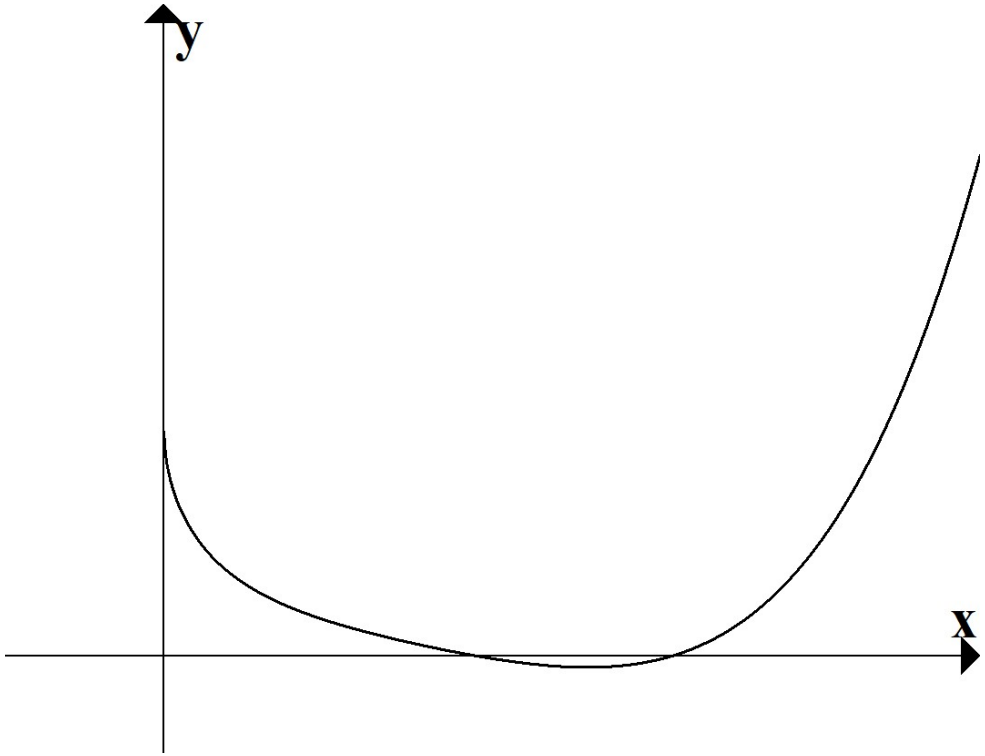
$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

***Entry Task*:** Consider the function

$$f(x) = (x^3 - 2x + 1)\left(3 + 5x - 6\sqrt{x}\right)$$

a. Find $f'(x)$.
b. Find the *height* at x = 1.
c. Find the *slope* at x = 1.
d. Give the *equation of the tangent* line at x = 1.

$$f(x) = (x^3 - 2x + 1)\left(3 + 5x - 6\sqrt{x}\right)$$



9.7 Chain Rule / Combining Rules

Generalized Power rule:

$$\frac{d}{dx}\Big(\big(g(x)\big)^n\Big) = n\big(g(x)\big)^{n-1}g'(x)$$

CHAIN RULE:

$$\frac{d}{dx}(f(g(x))) = f'\big(g(x)\big)g'(x)$$

Combining Rules

Step 0: Simplify and rewrite powers.

$$\frac{1}{x^r} = x^{-r}, \quad \sqrt[n]{x} = x^{1/n}$$

Step 1: Identify *overall* form

Sum: $A + B$

Product: $F \cdot S$

Quotient: $\frac{N}{D}$

Chain: $(\text{inside})^n$

Step 2: Apply rule.

As part of that rule, you likely will have to do more derivative. For those derivatives go to step 1.

Practice: Find the derivatives

Several of these are directly from HW!

1. $y = \dfrac{5\sqrt{1-x^3}}{9}$

2. $y = \dfrac{(x^2+2)^3}{x^4+5x}$

3. $y = \left(\frac{2x-4}{x^3+1}\right)^5$

4. $y = \frac{5}{x^3} + 6x^2\sqrt{x^5+1}$

5. $y = \dfrac{7}{2(x^4 + 8)^5} - 5x + 4$

6. $y = (t^2 + 4)^5 (t^3 - 2)^4$

9.8 Second Derivative

The *second derivative* is the derivative of the derivative. We denote it

$$f''(x) \quad \text{or} \quad \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

Example:

$$y = x^3 + 2x$$

The second derivative represents the rate at which the *rate* of the original quantity is changing.
(*rate of change of rate of change*)

We will interpret what this means more later, for now compute it.

Example: Suppose

$$R(x) = 70x + 0.4x^3$$

is the revenue (in dollars) if you sell x items.

1. What is the marginal revenue (denoted *MR* or $\overline{MR}$) when you sell 100 items?

2. What is the rate of change of marginal revenue when you sell 100 items?